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DAMPER WINDING REACTANCE.(U)

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DAMPER WINDING REACTANCE

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AFIT/GE/EE/78-5

Richard E. Moore
2 Lt. USAF

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Preface

This thesis was undertaken to develop a simple method for calculating damper winding reactances. Many engineers have published papers developing and presenting methods for calculating the reactances of the synchronous machine, however these papers are written for the engineer who is an expert in this field. These methods require the calculation or knowledge of fluxes, induced currents, voltages and/or other factors. This thesis reduces these methods to equations relating back to machine geometries only, allowing anyone with the geometries to calculate the damper winding reactances of the machine.

I would like to thank Dr. F. Brockhurst for his patience and guidance. I would also like to thank my wife for her guidance in the area of grammer and phraseology, and her tolerance of me during the writing.

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Abstract

A set of equations for calculating the reactance and resistance of synchronous machine damper windings is developed and presented. The final equations are in terms of machine geometries, allowing the user to calculate impedances without knowing fluxes,induced currents or voltages.

The derivation of this set of equations is based on the method developed by M. E. Talaat. In this derivation, equations for currents and induced voltages are developed, and then reduced to impedance ratios in terms of machine geometries only. From these ratios the reactances and resistances are found.

I. Introduction

Background

A considerable amount of publication has been presented in the area of calculating the performance of synchronous machines both in transient and steady state conditions. Scattered throughout this literature are sections developing the reactances of the synchronous machine and how they relate to the aforementioned conditions. The first major article in this area was written by C. J. Fechheimer (Ref 4) on self-starting synchronous motors in 1912, and was used as a basis for a number of papers through the late 1920's and early 1930's.

Through this period most of the work was done in the area of starting performance. Two papers which deal with reactances and how they relate to starting performance are by H. V. Putman (Ref 14) and T. M. Linville (Ref 9). Articles pertaining directly to the reactances of synchronous machines from this time period are more difficult to find, but a few include papers by P. L. Alger (Ref 1) and L. A. Kilgore (Ref 7) and a paper on the theory of synchronous machines by R. H. Park (Ref 13).

Since the middle of the 1930's there has been a decrease in the amount of published work in this area, however, a large portion of what has been published has been directly relevant to the reactance of the synchronous machine. G. C. Jain (Ref 6) and C. Concordia (Ref 2) have written books dealing completely with the synchronous machine, while A. S. Langsdorf (Ref 8), M. Liwschitz-Garik (Ref 11, Ref 12) and others have devoted sections of their books to this subject. Articles published during this time include one by H. C. DeJong (Ref 3) on the

starting performance of synchronous motors and two by M. E. Talaat (Ref 15, Ref 16) dealing directly with synchronous machine reactances.

Problem Statement

For the synchronous machine design process it is necessary for Air Force Aero Propulsion Laboratory, High Power Branch, to be able to calculate the damper winding reactances for various machine geometries. This would then become a part of their iterative design process used with alternating current machines. Presently, however, there is no simple way of calculating the reactances of the damper windings unless one is extremely familiar with the literature in this field.

This thesis presents an easy, straightforward method of developing and calculating these reactances from machine geometries.

Assumptions

1. Saturation, hysteresis and eddy currents are neglected.

2. The space distribution of the mmf by the armature winding is sinusoidal.

3. Symmetry is assumed about the center of the rotor pole and armature winding.

4. Assume only the 0^{th} and 2^{nd} permeance harmonics are present and the magnitude of the 2^{nd} harmonic of armature self and mutual inductance are equal.

5. The variation of the mutual inductances between any armature phase and the rotor circuit with respect to the rotor position is sinusoidal.

6. Reactances of the rotor circuits are independent of the rotor position.

7. The effective armature winding flux linkages are produced by the fundamental component of the air gap flux density.

8. The end rings are continuous and no damper bar slot openings face the interpole space.

Outline

The object of this thesis is not to develop new and unique equations, but rather to use material previously presented in the literature to develop a simple method for calculating the reactances of the damper windings. The development of the damper winding reactance equations is presented in Chapter II. This development uses basic principles, allowing the reader to understand the derivation of these equations. References are also provided for further clarification. Chapter III then presents the equations developed in Chapter II in a logical order for calculating the reactances from machine geometries.

The appendices contribute to the material covered in this thesis. All nomenclature from Chapters II and III is given in Appendix A, including a separate list of the machine geometries necessary for calculating the reactances. Appendix B presents a computer flow chart and FORTRAN program source listing to calculate the reactances. Included in the source listing are comment cards showing the input order of the machine geometries. Appendix C presents changes necessary in the equations to calculate the reactances of the damper windings referred to the rotor side.

II. Equation Development for Damper Winding Reactance

There have been many papers published on the reactances and starting performance of synchronous machines, but unless the reader has a clear understanding of the machines, he can easily become lost in the theory and even in the final equations. To alleviate some of this confusion, a straightforward method for the development of equations for calculating the damper winding reactances from machine geometries is presented. The development of the equations follows the method developed by M. E. Talaat (Ref 15, Ref 16), with clarification from other authors.

Calculation of the Mmf Due to the Equivalent Damper Bar Winding (Ref 15:179)

Currents of equal magnitude and opposite direction will be induced in any two damper bars equidistant from the pole center by either the direct or quadrature flux. The pair of damper windings farthest from the center of the pole will have maximum current. Let the current induced at 90 electrical degrees from the pole center be given as I_{Dd}. The current induced in the n^{th} damper bar pair would be

$$I_n = I_{Dd} \sin\left(\frac{\theta_n}{2}\right) \tag{1}$$

where θ_n is the electrical angle between the n^{th} pair of damper bars. The fundamental component of the mmf due to this current is

$$M_{Nd} = \frac{4}{\pi} I_n \sin\left(\frac{\theta_n}{2}\right) \tag{2}$$

amp turns per pole per bar pair. The resulting mmf due to all damper bar pairs is

$$M_{Nd} = \frac{4}{\pi}\left[\sum_{n=1}^{N_{Dd}} I_n \sin\left(\frac{\theta_n}{2}\right)\right] \tag{3}$$

where N_{Dd} is the number of damper bar pairs. Replacing I_n in Eq (3) by Eq (1) yields

$$M_{Dd} = \frac{4}{\pi} I_{Dd} \left[\sum_{n=1}^{N_{Dd}} \sin^2\left(\frac{\Theta_n}{2}\right) \right] \tag{4}$$

which can be reduced to

$$M_{Dd} = \frac{4}{\pi} I_{Dd} \left[N_{Dd} - \sum_{n=1}^{N_{Dd}} \cos \Theta_n \right] \tag{5}$$

Defining the uniform damper bar conversion factor as

$$A_{Dd} = \frac{1}{N_{Dd}} \left(\sum_{n=1}^{N_{Dd}} \cos \Theta_n \right) \tag{6}$$

Eq (5) becomes

$$M_{Dd} = \frac{2}{\pi} I_{Dd} N_{Dd} (1 - A_{Dd}) \tag{7}$$

If the damper bars are uniformly spaced and if n_b, the number of damper bars per pole, is odd, then

$$n_b = 2N_{Dd} + 1 \tag{8}$$

and

$$\Theta_1 = 2\alpha_b , \Theta_2 = 4\alpha_b \cdot \cdot \cdot \cdot \Theta_{N_{Dd}} = 2 N_{Dd} \alpha_b \tag{9}$$

where α_b is the electrical angle between two adjacent damper bars. If n_b is even, then

$$n_b = 2N_{Dd} \tag{10}$$

and

$$\Theta_1 = \alpha_b , \Theta_2 = 3\alpha_b \cdot \cdot \cdot \Theta_{N_{Nd}} = (2 N_{Dd} - 1) \alpha_b \tag{11}$$

If Eqs (9) and (11) are substituted into Eq (6) and this finite trigonometric series is reduced,

$$M_{Dd} = \frac{1}{\pi} n_b I_{Dd} (1 - k_b) \tag{12}$$

where

$$K_b = \frac{\sin n_b \alpha_b}{n_b \sin \alpha_b} = A_{Dd} \tag{13}$$

for n_b even. If n_b is odd, then

$$M_{Dd} = \frac{1}{\pi}(n_b - 1) I_{Dd}(1 - k_b) \tag{14}$$

where

$$K_b = \frac{\sin n_b \alpha_b - \sin \alpha_b}{(n_b - 1)\sin \alpha_b} = A_{Dd} \tag{15}$$

The equations for the amplitude of the damper bar mmf in the quadrature axis can be calculated by similar reasoning:

$$M_{Dq} = \frac{2}{\pi} I_{Dq} N_{Dd}(1 + A_{Dd}) \tag{16}$$

where

$$A_{Dd} = \frac{1}{N_{Dd}}\left(\sum_{n=1}^{N_{Dd}} \cos \theta_n\right) \tag{6}$$

for n_b even and

$$A_{Dd} = \frac{1}{N_{Dd}}\left(\sum_{n=1}^{N-1} \cos \theta_n + \frac{1}{2}\right) \tag{17}$$

for n_b odd.

For uniform spacing of the damper bars and with n_b even,

$$M_{Dq} = \frac{1}{\pi} I_{Dq} n_b (1 + k_b) \tag{18}$$

with k_b as defined in Eq (13). Similarly, with n_b odd,

$$M_{Dq} = \frac{1}{\pi} I_{Dq}(n_b - 1)(1 + k_b) \tag{19}$$

where

$$k_b = \frac{\sin n_b \alpha_b}{(n_b - 1)\sin \alpha_b} = A_{Dd} \tag{20}$$

<u>Calculation of the Direct and Quadrature Damper Winding Reactances</u> (Ref 15:180)

The amplitude of the fundamental component of the armature mmf due to an equivalent direct axis damper winding circuit is

$$M_a = \frac{4}{\pi} \frac{N k_w}{P} \dot{i}_{Dd} \tag{21}$$

for the first term of the Fourier series for the stator mmf. N is the number of stator series turns per pole per phase, P is the number of

poles (twice the number of pole pairs), k_ω is the mmf reduction factor due to the stator winding and i_{Dd} is the current in one phase of the stator. Equations (7) and (21) can be set equal to each other and solved for a current ratio, since the mmf due to the rotor and the stator must be equal:

$$\frac{I_{Dd}}{i_{Dd}} = \frac{2Nk_\omega}{PN_{Dd}(1-A_{Dd})} \tag{22}$$

The voltage induced in the rotor by the "A" phase in the stator is

$$e_{Dd} = \frac{2\pi fN}{10^8 P}\Phi k_\omega \tag{23}$$

while the voltage induced in the stator by the rotor is

$$E_{Dd} = \frac{2\pi fN_{Dd}}{10^8}k_{Dd}\Phi \tag{24}$$

where k_{Dd} is the direct axis rotor breadth factor. Using Eqs (23) and (24), a voltage ratio in terms of machine geometries can be obtained because the Φ's are the same:

$$\frac{e_{Dd}}{E_{Dd}} = \frac{Nk_\omega}{PN_{Dd}k_{Dd}} \tag{25}$$

Multiplying Eq (22) by Eq (25) will result in an impedance ratio:

$$\frac{I_{Dd}e_{Dd}}{i_{Dd}E_{Dd}} = \frac{2(Nk_\omega)^2}{P^2N_{Dd}^2k_{Dd}(1-A_{Dd})} \tag{26}$$

The total direct damper winding leakage flux is given by

$$\begin{aligned}\Psi_{LDd} &= 2P\lambda_{bd}\ell\left(\sum_{n=1}^{N_{Dd}} I_n\right)\\ &= 2PN_{Dd}k_{Dd}I_{Dd}\lambda_{bd}\ell\end{aligned} \tag{27}$$

where λ_{bd} is the equivalent single bar leakage permeance and ℓ is the core length. The definition of reactance is flux linkages per unit current, hence

$$x_{LDd} = \frac{2\pi f \psi_{LDd}}{10^8 I_{Dd}} = P N_{Dd} k_{Dd} 2 x_{bd} \tag{28}$$

and the equivalent single bar leakage reactance is

$$x_{bd} = \frac{2\pi f \ell \lambda_{bd}}{10^8} \tag{29}$$

Referring Eq (28) to the stator side by Eq (26),

$$X_{LDd} = \frac{4(Nk_\omega)^2}{PN_{Dd}(1-A_{Dd})} x_{bd} = \frac{4(Nk_\omega)^2}{10^8 P} \frac{2\pi f \ell \lambda_{bd}}{N_{Dd}(1-A_{Dd})} \tag{30}$$

Following the same procedure for the quadrature axis,

$$x_{LDq} = P N_{Dd} k_{Dd} 2 x_{bd} \tag{31}$$

where

$$x_{bq} = \frac{2\pi f \ell \lambda_{bq}}{10^8} \tag{32}$$

Referring Eq (31) to the stator side yields

$$X_{LDq} = \frac{4(Nk_\omega)^2}{PN_{Dd}(1+A_{Dd})} x_{bq} = \frac{4(Nk_\omega)^2}{10^8 P} \frac{2\pi f \ell \lambda_{bq}}{N_{Dd}(1+A_{Dd})} \tag{33}$$

The flux linkage in phase "A" due to the current in the equivalent direct axis damper winding is given by

$$\psi_{aDd} = N_{Dd}(1-A_{Dd}) N k_\omega \left(\tfrac{2}{\pi} C_{d1} \lambda_a\right) \ell I_{Dd} \tag{34}$$

The stator-rotor permeance factor is

$$\lambda_a = \frac{0.8\pi D}{P g_e} \tag{35}$$

where D is the rotor diameter, g_e is the equivalent gap and C_{d1} is the ratio of the fundamental air gap flux density produced by a sinusoidally distributed mmf, whose axis is through the pole center, to the mmf which would be produced with a uniform air gap equal to the effective air gap. Using Eq (22) to remove the current from Eq (34),

$$X_{aDd} = \frac{2\pi f \psi_{aDd}}{10^8 i_{Dd}} \frac{2Nk_\omega i_{Dd}}{PN_{Dd}(1-A_{Dd})} = \frac{4f\ell(Nk_\omega)^2}{10^8 P} C_{d1} \lambda_a \tag{36}$$

Following similar procedure, the quadrature reactance is given by

$$X_{aDq} = \frac{4f\ell(Nk_\omega)^2}{10^8 P} C_{q1}\lambda_a \tag{37}$$

The self-reactance of the equivalent damper winding circuit can be calculated in the same manner as X_{aDd} and X_{LDd}:

$$X_{DDd} = \frac{2\pi f}{10^8} \frac{2(Nk_\omega)^2}{P^2 N_{Dd}^2 k_{Dd}(1-A_{Dd})} \left[\frac{PN_{Dd}^2}{2}(1-A_{Dd})k_{Dd}\left(\frac{1}{\pi}C_{d1}\lambda_a\right) + PN_{Dd}k_{Dd}2\lambda_{bd}\right]$$

$$= \frac{4f\ell(Nk_\omega)^2}{10^8 P}\left[\lambda_a C_{d1} + \frac{\pi\lambda_{bd}}{N_{Dd}(1-A_{Dd})}\right]$$

$$= X_{aDd} + X_{LDd} \tag{38}$$

For the quadrature axis

$$X_{DDq} = \frac{4f\ell(Nk_\omega)^2}{10^8 P}\left[\lambda_a C_{q1} + \frac{\pi\lambda_{bq}}{N_{Dd}(1-A_{Dd})}\right]$$

$$= X_{aDq} + X_{LDq} \tag{39}$$

__Calculation of the Equivalent Resistance__ (Ref 15:181)

The power loss in the damper windings can be expressed as

$$P_{Dd} = 2r_{bd}I_{Dd}\left(\sin^2\frac{\theta_1}{2} + \sin^2\frac{\theta_2}{2} + \sin^2\frac{\theta_3}{2} + \cdots + \sin^2\frac{\theta_{N_{Dd}}}{2}\right)P \tag{40}$$

Using Eq (6) to reduce the finite series in Eq (40)

$$P_{Dd} = r_{bd}I_{Dd}^2 N_{Dd}(1-A_{Dd})P \tag{41}$$

and using Eq (22) to replace I_{Dd}

$$P_{Dd} = \frac{4(Nk_\omega)^2}{PN_{Dd}(1-A_{Dd})} i_{Dd}^2 r_{bd} \tag{42}$$

Dividing P_{Dd} by i_{Dd}^2 results in the damper winding resistance:

$$\frac{P_{Dd}}{i_{Dd}^2} = R_{Dd} = \frac{4(Nk_\omega)^2}{PN_{Dd}(1-A_{Dd})} r_{bd} \tag{43}$$

For the quadrature axis

$$R_{Dq} = \frac{4(Nk_\omega)^2}{PN_{Dd}(1+A_{Dd})} r_{bq} \tag{44}$$

Calculation of the Single Bar Leakage Permeance

Leakage permeances are usually calculated by determing the individual leakage permeances: slot leakage permeance, toothtip permeance, damper bar pitch permeance and end ring permeance; and summing them. The permeance is calculated for the reluctance of the air and copper and it is assumed that the reluctance of the iron is zero.

The permeance of path 1 in Fig. 1a and 1b is

$$.4\pi \frac{h_{b2}}{b_{b2}} \tag{45}$$

The remainder of the bar slot leakage permeance reduces to the factor

$$.4\pi (0.66) \tag{46}$$

for the round bar in Fig. 1b. The permeance of paths 2 and 3 in Fig. 1a is calculated as

$$.4\pi \left(\frac{h_{b4}}{b_{b1}} + \frac{2 h_{b2}}{b_{b1} + b_{b2}} \right) \tag{47}$$

In as much as the flux distribution through the damper bar is parabolic,

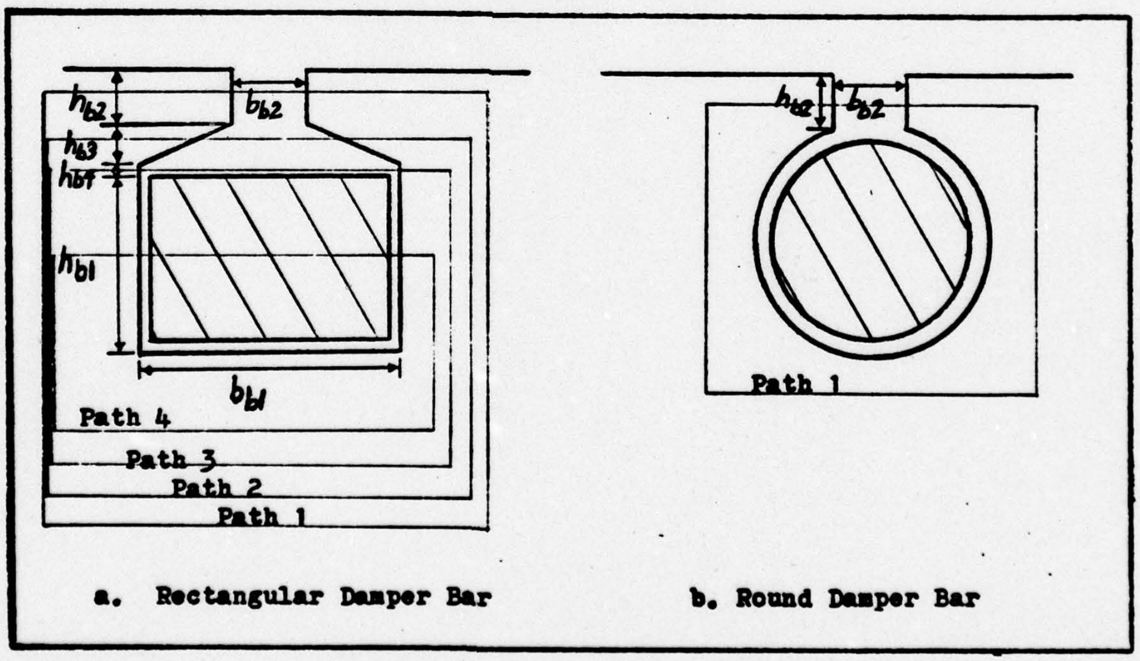


Fig. 1. Slot Leakage Permeance

the permeance of path 4 is

$$.4\pi \frac{h_{b1}}{3\, b_{b1}} \tag{48}$$

The toothtip leakage permeance is given by

$$.4\pi F_{ttd} \tag{49}$$

By the Schwarz-Christoffel Transform, F_{ttd} is given as (Ref 16:324)

$$F_{ttd} = 0.2284 + 0.0796 \frac{g}{b_s} - .25 \frac{b_s}{g}(1-\sigma) \tag{50}$$

where g is the gap under the pole center, b_s is the width of a stator slot and

$$\sigma = \frac{2}{\pi}\left[\arctan \frac{b_s}{2g} - \frac{g}{b_s} \log_e\left(1 + \frac{b_s^2}{4g^2}\right)\right] \tag{51}$$

For the quadrature axis

$$F_{ttq} = 0.2164 + 0.3183\left(\frac{b_t}{b_s}\right)^{.5} \tag{52}$$

where b_t is the width of a stator tooth.

The leakage permeance due to the damper bar pitch is (Ref 16:325, Ref 8:311)

$$.4\pi \frac{\tau_b}{12 g_e} \tag{53}$$

where
$$\tau_b = \frac{\tau}{n_b} \tag{54}$$

The leakage permeance due to the end rings is (Ref 16:323)

$$\frac{.4\pi}{m}\left[\frac{2n_b}{3} \frac{(l_b - l_h)}{l} + \frac{0.12\, \tau_r \tau_{ring}}{\tau_b l} Q\right] \tag{55}$$

where
$$\tau_r = \frac{\pi D}{P} \tag{56}$$

$$\tau_{ring} = \frac{\pi D'}{P} \tag{57}$$

and
$$Q = 2 + \frac{\cos n_b\alpha_b - k_b \cos \alpha_b}{(1-k_b)} \tag{58}$$

for the direct axis and

$$Q = \frac{\pi(1-\cos n_b\alpha_b)}{n_b\alpha_b(1+k_b)} + \frac{\cos n_b\alpha_b - k_b\cos\alpha_b}{(1+k_b)} \tag{59}$$

for the quadrature axis.

Reducing Eqs (45), (48) and (49) to per unit length and combining terms, the leakage permeance in the direct axis is

$$\lambda_{bd} = .4\pi\left[\frac{l_h}{l}\left(F_{ttd} + \frac{h_{b2}}{b_{b2}} + \frac{h_{b1}}{3b_{b1}} + \frac{h_{b4}}{b_{b1}} + \frac{2h_{b2}}{b_{b1}+b_{b2}}\right) + \frac{\tau_b}{12g_e} + \frac{2n_b}{3m}\left(\frac{l_b - l_h}{l}\right)\right.$$
$$\left. + \frac{0.12\pi^2 D_r D'}{m\tau_b l P^2}\left(2 + \frac{\cos n_b\alpha_b - k_b\cos\alpha_b}{(1-k_b)}\right)\right] \tag{60}$$

and in the quadrature axis

$$\lambda_{bq} = .4\pi\left[\frac{l_h}{l}\left(F_{ttq} + \frac{h_{b2}}{b_{b2}} + \frac{h_{b1}}{3b_{b1}} + \frac{h_{b4}}{b_{b1}} + \frac{2h_{b2}}{b_{b1}+b_{b2}}\right) + \frac{\tau_b}{12g_e} + \frac{2n_b}{3m}\left(\frac{l_b - l_h}{l}\right)\right.$$
$$\left. + \frac{0.12\pi^2 D_r D'}{m\tau_b l P^2}\left(\frac{\pi(1-\cos n_b\alpha_b)}{n_b\alpha_b(1+k_b)} + \frac{\cos n_b\alpha_b - k_b\cos\alpha_b}{(1+k_b)}\right)\right] \tag{61}$$

For the round damper bar, replace the term $\frac{h_{b1}}{3b_{b1}} + \frac{h_{b4}}{b_{b1}} + \frac{2h_{b2}}{b_{b1}+b_{b2}}$ with the factor 0.66.

All numerical factors are given for the cgs system. To convert all reactance equations to the English (inch) system, simply multiply Eqs (35), (60) and (61) by 2.54 centimeters/inch.

Calculation of the Stator Winding Factors

The Distribution Factor. (Ref 8:183) For a full pitch winding, as shown in Fig. 2a, the emf due to the coils can be expressed as

$$E_T = E_1\left(1 + e^{j\gamma} + e^{2j\gamma} + \cdots + e^{(q_1-1)j\gamma}\right) \tag{62}$$

where γ is the distance between slots and q_1 is the number of slots. Equation (62) can be reduced to

$$E_T = \frac{E_1 \sin\left(\frac{q\gamma}{2}\right)}{q\sin\left(\frac{\gamma}{2}\right)} \tag{63}$$

where
$$q = \frac{q_1}{m} \tag{64}$$

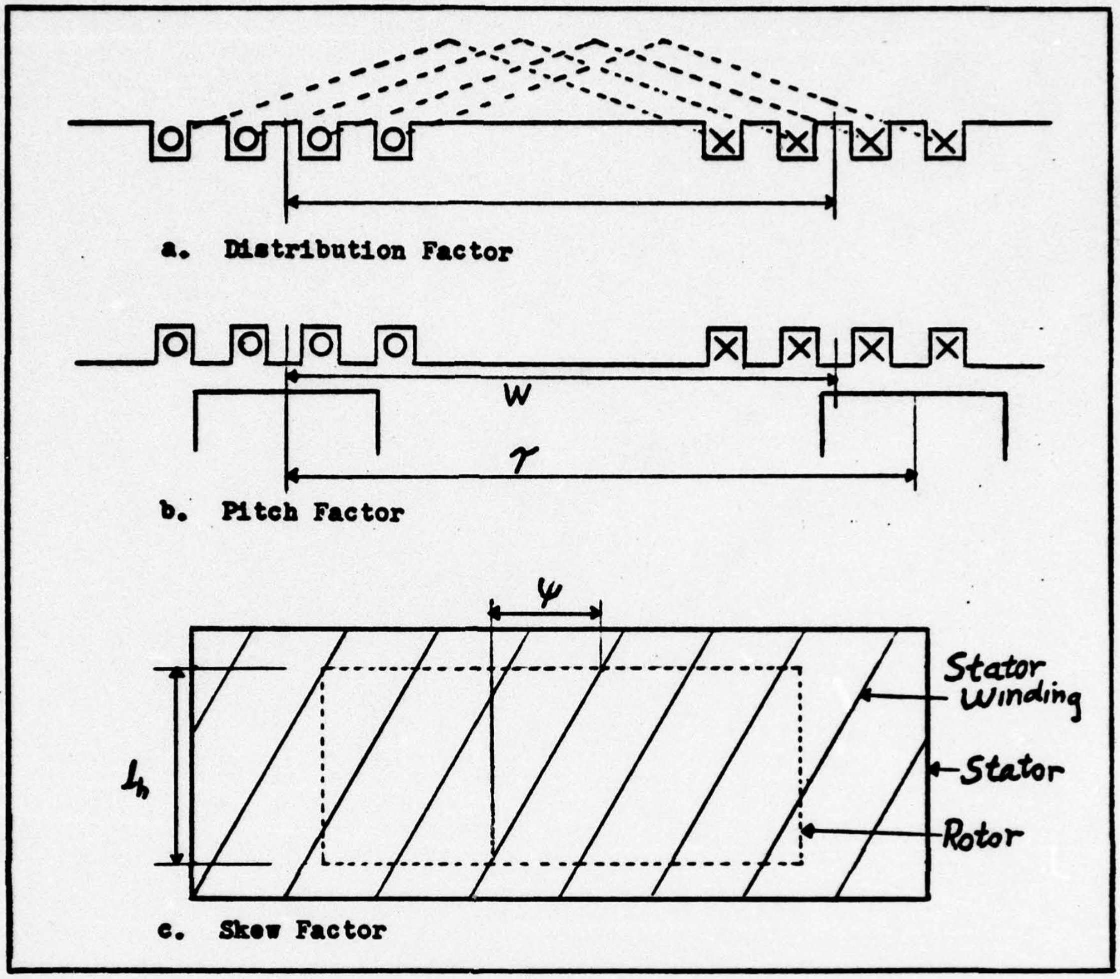


Fig. 2. Stator Distribution Factors

q is the number of slots per pole per phase, m is the number of phases and

$$\gamma = \frac{\pi}{qm} \tag{65}$$

The distribution factor, k_d, is the ratio of E_T to E_1:

$$k_d = \frac{E_T}{E_1} = \frac{\sin \frac{\pi}{2m}}{q_1 \sin \frac{\pi}{2mq}} \tag{66}$$

The Pitch Factor. (Ref 8:187,311) The pitch factor must be used to compensate for the partial pitch, since not all windings have 180° pitch (see Fig. 2b). This correction factor is given by

$$k_p = \sin \frac{W\pi}{2\tau} \quad (67)$$

where W is the stator pole width and τ is the rotor pole pitch.

The Skew Factor. (Ref 8:194) The effect of the stator winding is reduced by skewing the winding (see Fig. 2c). This factor is given by

$$K_s = \frac{\sin\left(\frac{\psi\pi}{2\ell_h}\right)}{\frac{\psi\pi}{2\ell_h}} \quad (68)$$

where ψ is the stator skew and ℓ_h is the rotor pole head length.

Calculation of the C_{d1} and C_{q1} Factors (Ref 5:197,219,223)

Calculation of C_{d1} and C_{q1} are done by curve fitting from flux plots (Ref 17):

$$C_{d1} = \beta_1 k_{ad} \quad (69)$$

and

$$C_{q1} = \beta_1 k_{aq} \quad (70)$$

where

$$\beta_1 = \left(\alpha + \frac{\sin \alpha\pi}{\pi}\right) \quad (71)$$

with α being the ratio of pole arc to pole pitch. For a constant air gap

$$k_{ad} = \frac{\alpha\pi + \sin \alpha\pi}{4 \sin \frac{\alpha\pi}{2}} \quad (72)$$

and

$$k_{aq} = \frac{\alpha\pi - \sin \alpha\pi}{4 \sin \frac{\alpha\pi}{2}} \quad (73)$$

while for a sinusoidal air gap

$$k_{ad} = \frac{4 \sin\left(\frac{\alpha\pi}{2}\right)\left[\cos^2\left(\frac{\alpha\pi}{2}\right) + 2\right]}{3\left[\alpha\pi + \sin(\alpha\pi)\right]} \quad (74)$$

and

$$k_{aq} = \frac{4 \sin^3\left(\frac{\alpha\pi}{3}\right)}{3\left[\alpha\pi + \sin(\alpha\pi)\right]} \quad (75)$$

Calculation of the Damper Winding Resistance

The resistance of a bar is given as

$$r_b = \rho \frac{\ell_b}{A_b} \tag{76}$$

where ρ is the resistivity of the material, ℓ_b is the length of the bar and A_b is the area of the bar. The resistance of the equivalent end rings is, by averaging the resistance of the end rings between each pair of bars, for the direct and quadrature axes, is

$$r_{rd} = \left(\frac{\sum_{n=1}^{N_{Dd}} \tau_{en}}{N_{Dd}} \right) \frac{\rho}{A_r}$$

and

$$r_{rq} = \left(\tau - \frac{\sum_{n=1}^{N_{Dd}} \tau_{en}}{N_{Dd}} \right) \frac{\rho}{A_r} \tag{77}$$

where τ_{en} is the pitch of the n^{th} end ring circuit and A_r is the area of the end ring. These equations for the direct and quadrature axes, using copper as the conductor, become

$$r_{bd} = \left(\frac{\ell_b}{A_b} + \frac{\sum_{n=1}^{N_{Dd}} \tau_{en}}{A_r N_{Dd}} \right) \frac{4.38}{10^{10}} \tag{78}$$

and

$$r_{bq} = \left(\frac{\ell_b}{A_b} + \frac{\tau}{A_r} - \frac{\sum_{n=1}^{N_{Dd}} \tau_{en}}{A_r N_{Dd}} \right) \frac{4.38}{10^{10}} \tag{79}$$

The preceding equations were developed for the reactances of the damper windings. The equations have been developed from machine voltages, currents and fluxes, but the final equations reduce to terms with machine geometriës only. This converts the equations to terms which the user can handle without having a vast knowledge in the fields of electromagnetics and machinery.

III. Presentation of Damper Winding Equations

Chapter II developed equations for the calculation of the damper winding reactances referred to the stator side. This chapter presents these equations in terms of machine geometries in a systematic order for calculating the reactances. Equations are presented for uniform and nonuniform damper bar spacing, as well as for round and rectangular damper bars.

Machine geometries used in these equations are in terms of centimeters. To change all equations to inch measurements, multiply Eqs (82), (83), (98) and (99) by 2.54 cm/in and divide Eqs (109) and (111) by the same factor.

<u>The Equations</u>

The equivalent single circuit damper bar leakage reactance (from Eqs (30) and (33)) in the direct and quadrature axes are given by

$$X_{LDd} = \frac{C_a \ell 2\pi f \lambda_{bd}}{10^8 C_{bd}} \tag{80}$$

and

$$X_{LDq} = \frac{C_a \ell 2\pi f \lambda_{bq}}{10^8 C_{bq}} \tag{81}$$

The amplitude of mutual reactances between the stator winding and the equivalent single circuit damper winding (from Eqs (36) and (37)) are calculated from

$$X_{aDd} = \frac{C_a \ell f (0.8\pi) D C_{d1}}{10^8 g_e} \tag{82}$$

and

$$X_{aDq} = \frac{C_a \ell f (0.8\pi) D C_{q1}}{10^8 g_e} \tag{83}$$

The self-reactances of the equivalent single circuit damper winding are

$$X_{DDd} = X_{LDd} + X_{aDd} \tag{38}$$

and $$X_{DDq} = X_{LDq} + X_{aDq} \tag{39}$$

The resistance of the damper windings (from Eqs (43) and (44)) is given as

$$R_{Dd} = \left(\frac{C_a}{C_{bd}}\right) r_{bd} \tag{84}$$

$$R_{Dq} = \left(\frac{C_a}{C_{bq}}\right) r_{bq} \tag{85}$$

Terms from the previous set of equations are listed below, with, if necessary, a reference to where further evaluation of that term can be found:

C_a	Stator-rotor common factor Eq (86)
C_{bd}	Equivalent direct axis damper bar conversion factor Eqs (87), (91), (96)
C_{bq}	Equivalent quadrature axis damper bar conversion factor Eqs (88), (92), (97)
C_{d1}	Direct axis pole factor Eq (107)
C_{q1}	Quadrature axis pole factor Eq (108)
D	Stator-rotor bore
f	Supply frequency
g_e	Equivalent gap Eq (104)
ℓ	Core length Fig. 5b
P	Number of poles or twice the number of pole pairs
r_{bd}	Equivalent direct axis damper bar resistance Eq (109)
r_{bq}	Equivalent quadrature axis damper bar resistance Eq (111)
λ_{bd}	Direct axis equivalent single bar leakage permeance Eq (98)
λ_{bq}	Quadrature axis equivalent single bar leakage permeance Eq (99)

The stator-rotor common factor, which is used in all the reactance equations, is

$$C_a = \frac{4(Nk_d k_p k_s)^2}{P} \tag{86}$$

where
$$k_d = \frac{\sin\left(\frac{\pi}{2m}\right)}{q \sin\left(\frac{\pi}{2mq}\right)} \tag{66}$$

$$k_p = \sin\left(\frac{W\pi}{\tau 2}\right) \tag{67}$$

and
$$K_s = \frac{\sin\left(\frac{\psi\pi}{2\ell_h}\right)}{\frac{\psi\pi}{2\ell_h}} \tag{68}$$

Terms used in the above equations are

k_d	Stator distribution factor Eq (66)
k_p	Stator pitch factor Eq (67)
k_s	Stator skew factor Eq (68)
ℓ_h	Rotor pole head length Fig. 5b
m	Number of phases
N	Number of stator series turns per pole per phase
q	Number of stator slots per pole per phase
W	Stator coil width Fig. 5a
τ	Pole pitch Fig. 5a
ψ	Stator winding skew Fig. 5b

__Calculation of the Equivalent Damper Bar Conversion Factor__

From Eqs (7) and (16), the equivalent damper bar conversion factor can be calculated as

$$C_{bd} = N_{Dd}(1 - A_{Dd}) \tag{87}$$

and
$$C_{bq} = N_{Dd}(1 + A_{Dd}) \tag{88}$$

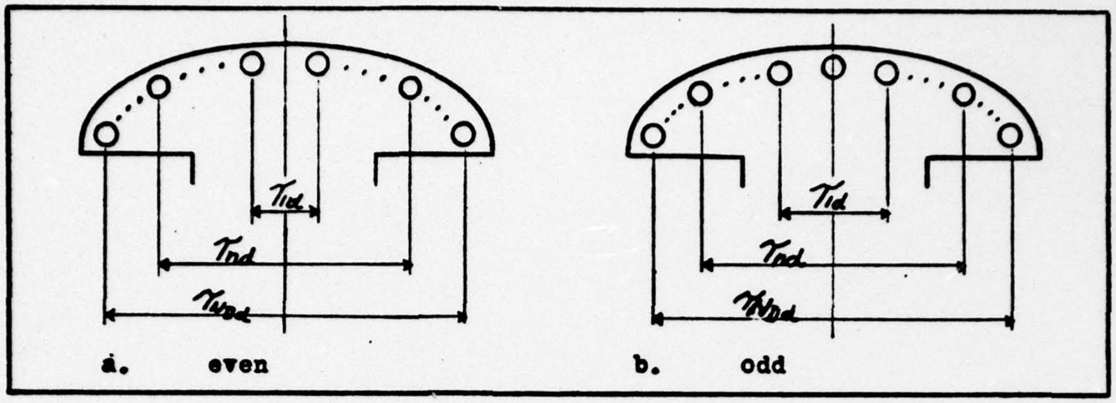


Fig. 3. Pitch of the Damper Bar Circuits

The calculation of N_{Dd}, the number of damper bar pairs, and A_{Dd}, the uniform damper bar conversion factor, differ due to whether the number of damper bars, n_b, is even or odd.

n_b **Even**. In the case where n_b is even, A_{Dd} in the direct and quadrature axes is calculated by (from Eq (6))

$$A_{Dd} = \frac{1}{N_{Dd}}\left[\sum_{n=1}^{N_{Dd}} \cos\left(\frac{\tau_{nd}}{\tau}\pi\right)\right] \tag{89}$$

and

$$N_{Dd} = \frac{n_b}{2} \tag{90}$$

where τ_{nd} is the damper bar pitch of the n^{th} damper bar circuit, as shown in Fig. 3. For uniform spacing between damper bars, k_b will be used in place of A_{Dd} and

$$C_{bd} = \frac{n_b}{2}\left(1 - k_b\right) \tag{91}$$

$$C_{bq} = \frac{n_b}{2}\left(1 + k_b\right) \tag{92}$$

where

$$k_b = \frac{\sin n_b \alpha_b}{n_b \sin \alpha_b} \tag{93}$$

$$N_{Dd} = \frac{n_b}{2} \tag{90}$$

and

$$\alpha_b = \frac{\tau_{1d}}{\tau}\pi \tag{93}$$

α_b is the angle between two adjacent damper bars.

n_b Odd. For n_b odd, A_{Dd} (from Eq (6)) in the direct axis is given by

$$A_{Dd} = \frac{1}{N_{Dd}}\left[\sum_{n=1}^{N_{Dd}} \cos\left(\frac{T_{nd}}{T}\pi\right)\right] \tag{89}$$

where

$$N_{Dd} = \frac{n_b - 1}{2} \tag{94}$$

In the quadrature axis

$$A_{Dd} = \frac{1}{N_{Dd}}\left[\sum_{n=1}^{N_{Dd}} \cos\left(\frac{T_{nd}}{T}\pi\right) + \frac{1}{2}\right] \tag{95}$$

For the case of uniform damper bar spacing, in the direct axis the equivalent damper bar conversion factor is

$$C_{bd} = \left(\frac{n_b - 1}{2}\right)\left(1 - k_b\right) \tag{96}$$

where

$$k_b = \frac{\sin n_b\alpha_b - \sin\alpha_b}{(n_b - 1)\sin\alpha_b} \tag{15}$$

while for the quadrature axis

$$C_{bq} = \left(\frac{n_b - 1}{2}\right)\left(1 + k_b\right) \tag{97}$$

where

$$k_b = \frac{\sin n_b\alpha_b}{(n_b - 1)\sin\alpha_b} \tag{20}$$

<u>Calculation of the Equivalent Single Bar Leakage Permeance</u>

The equivalent single bar leakage permeance in the direct and quadrature axes is given by

$$\lambda_{bd} = .4\pi\left(k_1 + k_{2d}\right) \tag{98}$$

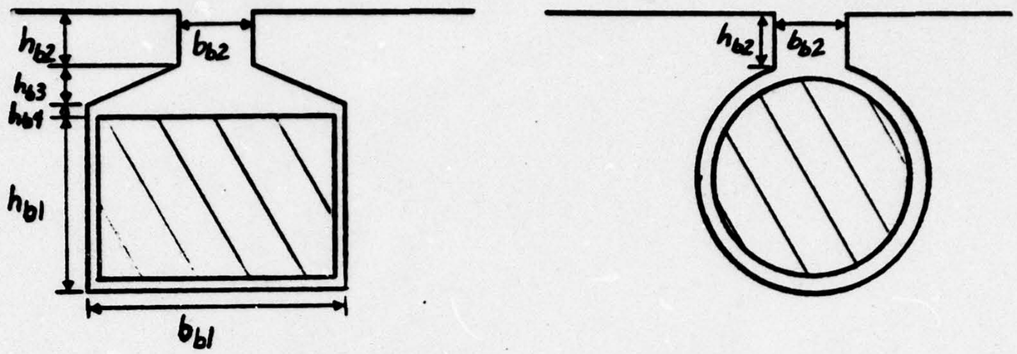
$$\lambda_{bq} = .4\pi\left(k_1 + k_{2q}\right) \tag{99}$$

where

$$k_1 = \frac{\ell_h}{\ell}\left(\frac{h_{b4}}{b_{b1}} + \frac{2h_{b2}}{b_{b1} + b_{b2}} + \frac{h_{b1}}{3b_{b1}} + \frac{h_{b2}}{b_{b2}}\right) + \frac{\tau_b}{12g_e} + \frac{2n_b}{3m}\cdot\frac{(\ell_b - \ell_h)}{\ell} \tag{100}$$

for rectangular damper bars (Fig. 4a). For round damper bars

$$k_1 = \frac{\ell_h}{\ell}\left(.66 + \frac{h_{b2}}{b_{b2}}\right) + \frac{\tau_b}{12g_e} + \frac{2n_b}{3m}\cdot\frac{(\ell_b - \ell_h)}{\ell} \tag{101}$$



a. Rectangular Damper Bar b. Round Damper Bar

Fig. 4. Slot Leakage Permeance Factors

Defining the equations for k_{2d} and k_{2q},

$$k_{2d} = \frac{.12\pi n_b D'}{m\ell P}\left(2 + \frac{\cos n_b\alpha_b - k_b\cos\alpha_b}{(1-k_b)}\right) + \frac{\ell_h}{\ell}F_{ttd} \qquad (102)$$

and

$$k_{2d} = \frac{.12\pi n_b D'}{m\ell P}\left(\frac{\pi(1-\cos n_b\alpha_b)}{n_b\alpha_b(1+k_b)} + \frac{\cos n_b\alpha_b - k_b\cos\alpha_b}{1+k_b}\right) + \frac{\ell_h}{\ell}F_{ttq} \qquad (103)$$

In these equations

b_{b1}	Width of the damper bar slot	Fig. 4a
b_{b2}	Width of the damper bar slot opening	Fig. 4a and 4b
D'	Mean diameter of the end rings	
D_r	Rotor diameter	
F_{ttd}	Bar slot toothtip leakage permeance in the direct axis	Eq (50)
F_{ttq}	Bar slot toothtip leakage permeance in the quadrature axis	Eq (52)
g_e	Equivalent gap	Eq (104)
h_{b1}	Depth of damper bar	Fig. 4a
h_{b2}	Depth of damper bar slot opening	Fig. 4a and 4b
h_{b3}	Depth of angled section of damper bar slot	Fig. 4a

h_{b4} Depth of damper bar within the damper bar slot Fig. 4a

ℓ Core length Fig. 5b

ℓ_b Damper bar length

ℓ_h Rotor pole head length Fig. 5b

α_b Average angle between adjacent damper bars

and
$$k_b = \frac{\sin n_b \alpha_b}{n_b \sin \alpha_b} \tag{13}$$

Repeating Eqs (50), (51) and (52) from Chapter II,

$$F_{ttd} = 0.2284 + 0.0796\left(\frac{g}{b_s}\right) - 0.25\,\frac{b_s}{g}(1-\sigma) \tag{50}$$

where
$$\sigma = \frac{2}{\pi}\left(\arctan\left(\frac{b_s}{2g}\right) - \frac{g}{b_s}\log_e\left(1 + \frac{b_s^2}{4g^2}\right)\right) \tag{51}$$

and
$$F_{ttq} = 0.2164 + 0.3183\left(\frac{b_t}{b_s}\right)^{.5} \tag{52}$$

The equivalent gap is given by

$$g_e = \left(\frac{\alpha^2 + (\rho_g - 1)}{\alpha^2}\right) g \tag{104}$$

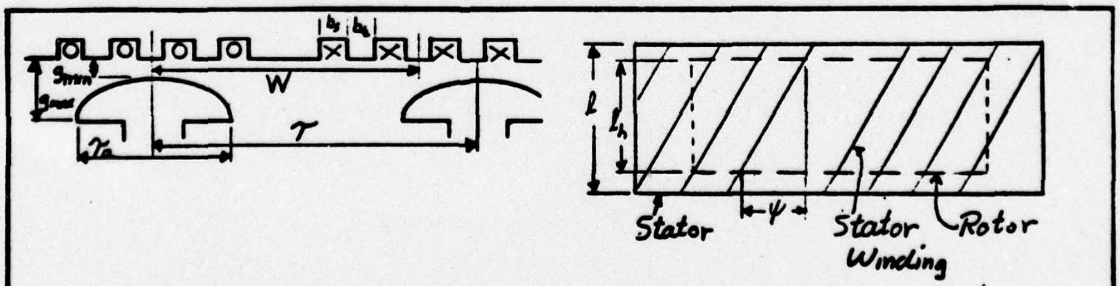
where
$$\alpha = \frac{\tau_a}{\tau} \tag{105}$$

$$\rho_g = \frac{g_{max}}{g_{min}} \tag{106}$$

b_s Width of the stator slot

b_t Width of the stator tooth

g Gap under pole center



a. Stator Rotor side view b. Stator Rotor top view

Fig. 5. Stator Winding Coefficients

g_{max}	**Maximum gap**	**Fig. 5a**
g_{min}	**Minimum gap**	**Fig. 5a**
τ	**Pole pitch**	**Fig. 5a**
τ_a	**Pole arc**	**Fig. 5a**

Calculation of C_{d1} and C_{q1}, the Direct and Quadrature Axes Pole Factors

From Eqs (69), (70) and (71)

$$C_{d1} = \left(\alpha + \frac{\sin\alpha\pi}{\pi}\right) k_{ad} \tag{107}$$

$$C_{q1} = \left(\alpha + \frac{\sin\alpha\pi}{\pi}\right) k_{aq} \tag{108}$$

where for a uniform gap

$$k_{ad} = \frac{\alpha\pi + \sin\alpha\pi}{4\sin\frac{\alpha\pi}{2}} \tag{72}$$

and

$$k_{aq} = \frac{\alpha\pi - \sin\alpha\pi}{4\sin\frac{\alpha\pi}{2}} \tag{73}$$

and for a sinusoidal gap

$$k_{ad} = \frac{4\sin\left(\frac{\alpha\pi}{2}\right)\left[\cos^2\left(\frac{\alpha\pi}{2}\right) + 2\right]}{3\left[\alpha\pi + \sin(\alpha\pi)\right]} \tag{74}$$

and

$$k_{aq} = \frac{4\sin^3\left(\frac{\alpha\pi}{3}\right)}{3\left[\alpha\pi + \sin(\alpha\pi)\right]} \tag{75}$$

Calculation of the Resistance of the Damper Bars

From Eq (78) the resistance of the damper bar and end ring in the direct axis is

$$r_{bd} = \left(\frac{l_b}{A_b} + \frac{\sum_{n=1}^{N_{Dd}} \tau_{en}}{A_r N_{Dd}}\right)\rho \tag{109}$$

where

$$\tau_{en} = \tau_{nd}\frac{D'}{D_r} \tag{110}$$

A_b Cross-sectional area of the damper bar

A_r Cross-sectional area of the end ring

D' End ring mean diameter

D_r Rotor diameter

l_b Length of the damper bar

N_{Dd} Number of damper bar circuits

ρ Resistivity of the damper bar and end ring material (copper is 4.38×10^{-10} ohms-cm)

τ Pole pitch

τ_{en} Pitch of the n^{th} end ring circuit

τ_{nd} Pitch of the n^{th} damper bar circuit

The resistance of the damper bar and end ring in the quadrature axis (from Eq (79)) is

$$r_{bq} = \left(\frac{l_b}{A_b} + \frac{\tau D'}{A_r D} - \frac{\sum_{n=1}^{N_{Dd}} \tau_{en}}{A_r N_{Dd}} \right) \rho \qquad (111)$$

Equations necessary for the calculation of damper winding reactance and resistance referred to the stator side have been presented in this chapter in a systematic order for evaluation. While there are many calculations necessary to determine the reactance, they are all in terms of machine geometries and with patience the user can determine the reactance and resistance of the damper windings.

IV. Conclusions and Recommendations

Equations for the calculation of the damper bar reactances and resistances have been developed and presented with reference to the stator. All equations reduce to machine geometries, allowing the user to evaluate the reactances without knowing flux densities, induced voltages or currents within the machine. To assist the user, Appendix A contains a list of nomenclature from Chapter II and Chapter III, plus a list of all the machine geometries necessary for calculating the equivalent circuit damper bar reactances, from the equations in Chapter III. Appendix B presents a computer flow chart as well as a FORTRAN source listing for the CDC 6600, with a list of the machine geometries needed to calculate the reactances and resistances of the damper windings. Appendix C presents the changes necessary to convert the the *reference of the* equations from the stator to the rotor.

Areas for Further Study

Whereas this thesis is complete in and of itself, there are some areas covered in the literature that are worthy of further study:

1. Devise a method to incorporate saturation into the reactance equations.

2. Develop equations for the reactances if the end rings are not continuous.

3. Develop equations for the reactances of a synchronous machine having damper bars in the pole head tips with slots facing the inter-pole space.

4. Develop equations to calculate the reactances of the higher harmonics.

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Appendix A

Nomenclature

Note: Some terms may have an additional d or q as a subscript referring to the direct or quadrature axis.

A_b	Cross-sectional area of damper bar
A_D	Uniform damper bar conversion factor
A_r	Cross-sectional area of end ring
b_{b1}	Width of damper bar slot
b_{b2}	Width of damper bar slot opening
b_s	Width of stator slot
b_t	Width of stator tooth
C_a	Stator-rotor common factor
C_b	Equivalent damper bar conversion factor
C_{d1}	Direct axis pole factor
C_{q1}	Quadrature axis pole factor
D	Stator-rotor bore
D'	End ring average diameter
D_r	Rotor diameter
e_D	Voltage induced in rotor by "A" phase
E_D	Voltage induced in stator by rotor equivalent damper winding
E_T	Total voltage induced by a winding
E_1	Voltage induced by a coil
f	Supply frequency
F_{tt}	Bar slot toothtip leakage permeance
g	Gap under pole center
g_e	Equivalent gap
g_{max}	Maximum gap
g_{min}	Minimum gap

h_{b1}	Depth of damper bar
h_{b2}	Depth of damper bar slot opening
h_{b3}	Depth of angled section of damper bar slot
h_{b4}	Depth of damper bar within the damper bar slot
i_D	Current in "A" phase of stator
I_D	Current in equivalent damper bar circuit
I_n	Current induced in the n^{th} damper bar pair
k_a	Saliency coefficient
k_b	Uniform spacing equivalent damper bar conversion factor
k_D	Rotor winding breadth factor
k_d	Stator distribution factor
k_p	Stator pitch factor
k_s	Stator skew factor
k_ω	Stator winding factor $(k_d k_p k_s)$
k_1	Common factor in the direct and quadrature permeance
k_2	End ring plus bar slot toothtip leakage permeance
ℓ	Core length
ℓ_b	Damper bar length
ℓ_h	Rotor pole head length
m	Number of phases
M_a	Mmf due to current in stator "A" phase
M_D	Mmf due to equivalent rotor damper bar circuit
M_N	Mmf due to current in n^{th} damper bar circuit
N	Number of stator series turns per pole per phase
n_b	Number of damper bars per pole
N_D	Number of damper bar circuits
P	Number of poles (twice the number of pole pairs)
P_D	Power loss in damper winding per pole

q	Number of stator slots per pole per phase
q_1	Number of stator slots
Q	Leakage factor in direct and quadrature axes
r_b	Equivalent single bar resistance
R_D	Resistance of equivalent single circuit damper winding
r_r	Resistance of the end rings
W	Stator coil width
x_{aD}	Amplitude of mutual reactances between the stator "A" phase winding and the equivalent single circuit damper winding--Rotor side
X_{aD}	Amplitude of mutual reactances between the stator "A" phase winding and the equivalent single circuit damper winding--Stator side
x_b	Equivalent single bar leakage reactance
X_{DD}	Self-reactance of the equivalent single circuit damper winding
X_{LD}	Equivalent single circuit damper leakage reactance
α	Ratio of pole arc to pole pitch
α_b	Average electrical angle between adjacent damper bars
β_1	Amplitude of the first harmonic divided by the amp-turns D.C.
γ	Distance between stator circuits (π/qm)
θ_n	Electrical angle between n^{th} pair of damper bars
λ_a	Stator-rotor leakage factor $(0.8\pi/P_{g_e})$
λ_b	Equivalent single bar leakage permeance
ρ	Resistivity of damper bar and end ring
ρ_g	Ratio of maximum gap to minimum gap
σ	Factor appearing in F_{tt}
τ	Pole pitch
τ_a	Pole arc
τ_b	Pole face damper winding bar pitch (τ/n_b)
τ_{en}	Pitch of the n^{th} end ring circuit $(\tau_{nd} D'/D_r)$

τ_{nd}	Pitch of the n^{th} damper bar circuit
τ_r	Stator pole pitch at stator-rotor bore $(\pi D/P)$
τ_{ring}	Pole pitch at average ring diameter $(\pi D'/P)$
Φ	Flux per pole
ψ	Stator winding skew
ψ_{aD}	Flux linkages in phase "A" due to current in the equivalent axis damper winding
ψ_{LD}	Damper winding leakage flux

<u>Machine Geometries Necessary for Calculating Reactances in Chapter III</u>

A_b	Cross-sectional area of damper bar
A_r	Cross-sectional area of end ring
b_{b1}	Width of damper bar slot
b_{b2}	Width of damper bar slot opening
b_s	Width of stator slot
b_t	Width of stator tooth
D	Stator-rotor bore
D'	End ring average diameter
D_r	Rotor diameter
f	Supply frequency
g	Gap under pole center
g_{max}	Maximum gap
g_{min}	Minimum gap
h_{b1}	Depth of damper bar
h_{b2}	Depth of damper bar slot opening
h_{b3}	Depth of angled section of damper bar slot
h_{b4}	Depth of damper bar within the damper bar slot

l	Core length
l_b	Damper bar length
l_h	Rotor pole head length
m	Number of phases
N	Number of stator series turns per pole per phase
n_b	Number of damper bars per pole
P	Number of poles (twice the number of pole pairs)
q	Number of stator slots per pole per phase
W	Stator coil width
α_b	Average electrical angle between adjacent damper bars
τ	Pole pitch
τ_a	Pole arc
τ_n	Pitch of the n^{th} damper bar circuit

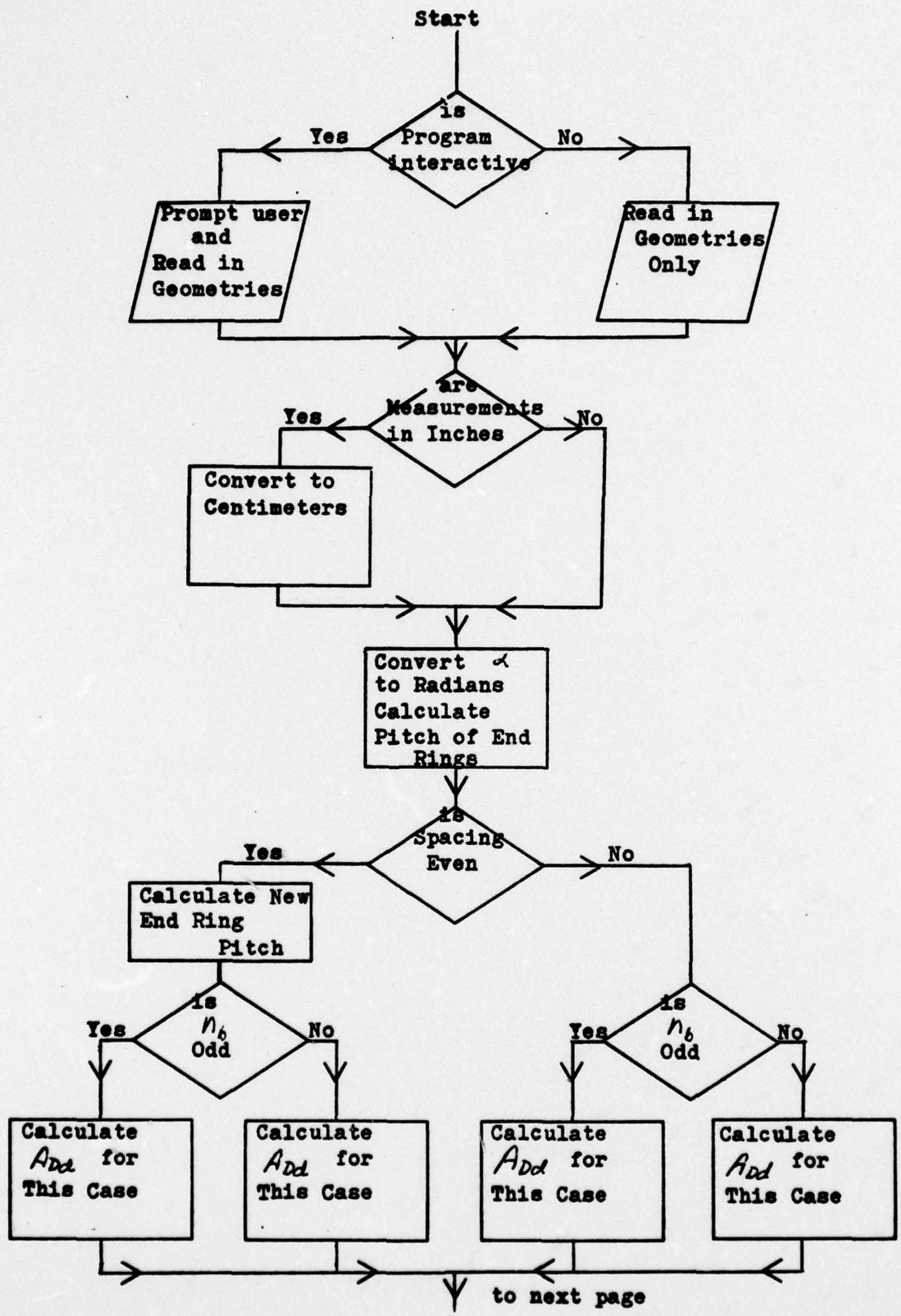
Appendix B

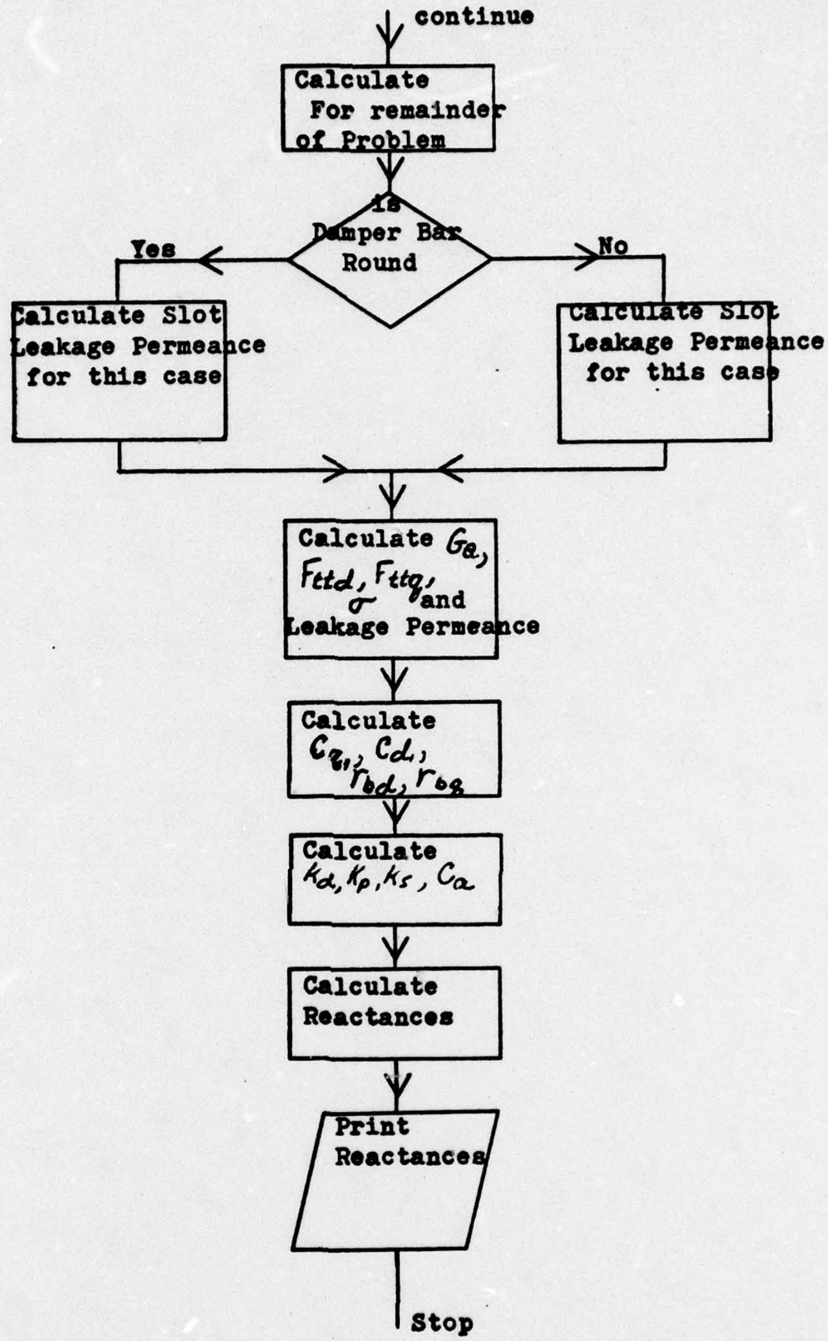
Computer Program

Flow Chart and

Source Listing

Flow Chart


Start
is Program interactive
Yes
No
Prompt user and Read in Geometries
Read in Geometries Only
are Measurements in Inches
Yes
No
Convert to Centimeters
Convert α to Radians Calculate Pitch of End Rings
is Spacing Even
Yes
No
Calculate New End Ring Pitch
is n_b Odd
Yes
No
is n_b Odd
Yes
No
Calculate A_{Dd} for This Case
Calculate A_{Dd} for This Case
Calculate A_{Dd} for This Case
Calculate A_{Dd} for This Case
to next page


continue
Calculate For remainder of Problem
Is Damper Bar Round
Yes
No
Calculate Slot Leakage Permeance for this case
Calculate Slot Leakage Permeance for this case
Calculate G_a, F_{ttd}, F_{ttq}, σ and Leakage Permeance
Calculate C_{q1}, C_d, r_{bd}, r_{bq}
Calculate k_d, k_p, k_s, C_a
Calculate Reactances
Print Reactances
Stop

Source Listing

```
      PROGRAM REACT(INPUT,OUTPUT.TAPE5)
C
C
C
C     FOR TTY SITE TYPE "1"; IF NOT IGNORE
C
C     FOR INCH MEASUREMENTS ENTER 1 FOR METRIC ENTER 2
C
C
C  INPUT VARIABLES---
C     LINE FREQUENCY, NUMBER OF PHASES, NUMBER OF POLES
C
C   MACHINE DIMENSIONS---
C     CORE LENGTH, ROTOR POLE HEAD LENGTH, DAMPER BAR LENGTH, STATOR-
C     ROTOR BORE, END RING DIAMETER, ROTOR DIAMETER
C
C   STATOR DIMENSIONS---
C     NUMBER OF STATOR SERIES TURNS PER POLE PER PHASE, NUMBER OF STATOR
C     SLOTS PER POLE PER PHASE, STATOR POLE WIDTH,STATOR SLOT WIDTH,
C     STATOR TOOTH WIDTH, STATOR WINDING SKEW
C
C   ROTOR DIMENSIONS---
C     ROTOR POLE PITCH AND ARC
C     IF THE SPACING BETWEEN DAMPER BARS IS UNIFORM ENTER A "1",IF NOT
C     ENTER A "2"
C     IF UNIFORM, ENTER THE PHYSICAL ANGLE BETWEEN TWO DAMPER BARS
C
C     IF NOT UNIFORM ENTER THE PITCHES BETWEEN THE DAMPER BAR CIRCUITS
C     GAP UNDER POLE CENTER MAXIMUM AND MINIMUM GAPS
C     IF ROUND DAMPER BAR ENTER A "1",IF NOT ENTER A "2"
C     FOR ROUND DAMPER BAR ENTER DAMPER BAR SLOT OPENING WIDTH AND DEPTH
C     FOR RECTANGULAR BAR ENTER DAMPER BAR SLOT AND SLOT OPENING WIDTH
C     BAR SLOT OPENING DEPTH ANGLED SECTION OF DAMPER BAR SLOT DEPTH,
C     DEPTH OF DAMPER BAR IN DAMPER BAR SLOT AND DEPTH OF DAMPER BAR
C     ENTER DAMPER BAR AND END RING CROSS-SECTIONAL AREAS
C
C
```

```
C  OUTPUT VARIABLES---
C      THE AMPLITUDE OF MUTUAL REACTANCES BETWEEN THE STATOR "A" PHASE
C      AND THE EQUIVALENT SINGLE CIRCUIT DAMPER WINDING
C      THE SELF REACTANCE OF THE EQUIVALENT SINGLE CIRCUIT DAMPER
C      WINDING
C      THE EQUIVALENT SINGLE CIRCUIT DAMPER LEAKAGE REACTANCE
C      AND
C      THE RESISTANCE OF THE EQUIVALENT SINGLE CIRCUIT DAMPER WINDING
C       FOR THE DIRECT AND QUADRATURE AXIS
C
C
C
C
       DIMENSION TNN(20),TRN(20)
       TNNT=ADD=0.
       PI=3.1415926
       WRITE 1001
       READ *,N
       IF(N.NE.10) GO TO 101
       WRITE 1002
       READ *,NIN
       WRITE 1003
       READ *,F,PH,PO
       WRITE 1004
       READ *,DL,DLH,DLB
       WRITE1005
       READ *,D,DE,DR
       WRITE 1006
       READ *,XN,XS
       WRITE 1007
       READ *,BS,BT,CHI
       WRITE 1008
       READ *,W,TA,TAA
       WRITE 1009
       READ *,NB
       NDD=NB/2
```

```
      ANB=FLOAT(NB)
      WRITE 1010
      READ *,NSP
      IF(NSP.NE.1) GO TO 98
      WRITE 1011
      READ *,ALP
      GO TO 97
   98 WRITE 1012,NDD,NDD
      READ *,(TNN(N),N=1,NDD)
   97 WRITE 1013
      READ *,GA,GMA,GMI
      WRITE 1014
      READ *,NRO
      IF(NRO.NE.1) GO TO 96
      WRITE 1015
      READ *,BB2,HB2
      GO TO 95
   96 WRITE 1016
      READ *,BB1,BB2
      WRITE 1017
      READ *,HB2,HB3,HB4,HB1
   95 WRITE 1018
      READ *,AB,AR
      GO TO 102
  101 NIN=N
      READ *,F,PH,PO,DL,DLH,DLB,D,DE,DR,XN,XS,BS,BT,CHI,W,TA,TAA,NB,NSP
      NDD=NB/2
      ANB=FLOAT(NB)
      IF(NSP.NE.1) GO TO 94
      READ *,ALP
      GO TO 93
   94 READ *,(TNN(N),N=1,NDD)
   93 READ *,GA,GMA,GMI,NRO
      IF(NRO.NE.1) GO TO 92
      READ *,BB2,HB2
      GO TO 91
```

```
92    READ *,BB1,BB2,HB2,HB3,HB4,HB1
91    READ *,AB,AR
102   CONTINUE
      CM=2.54
      IF(NIN.NE.1) GO TO 150
      DL=DL*CM
      DLH=DLH*CM
      DLB=DLB*CM
      D=D*CM
      DE=DE*CM
      DR=DR*CM
      BS=BS*CM
      BT=BT*CM
      CHI=CHI*CM
      W=W*CM
      TA=TA*CM
      TAA=TAA*CM
      DO 140 N=1,NDD
      TNN(N)=TNN(N)*CM
140   TRN(N)=TRN(N)*CM
      TNNT=TNNT*CM
      GA=GA*CM
      GMA=GMA*CM
      GMI=GMI*CM
      BB1=BB1*CM
      BB2=BB2*CM
      HB1=HB1*CM
      HB2=HB2*CM
      HB3=HB3*CM
      HB4=HB4*CM
      AB=AB*(CM*CM)
      AR=AR*(CM*CM)
150   CONTINUE
      ALP=ALP*PI*PO/360.
      APP=DE/DR
      DO 100 N=1,NDD
```

```
      TRN(N)=TNN(N)*ARP
100   TNNT=TNNT+TRN(N)
      TNNT=TNNT/FLOAT(NDD)
      ALPB=TAA/TA
1001   FORMAT(2X,"IF YOU ARE AT A TTY TERMINAL ENTER A .10.")
1002  FORMAT(2X,"IF YOUR MEASUREMENTS ARE IN INCHES, ENTER A .1. IF IN C
     1M ENTER A .2.")
1003  FORMAT(2X,"ENTER SUPPLY FREQUENCY,NUMBER OF PHASES AND POLES ")
1004  FORMAT(2X,"ENTER CORE,POLE HEAD, AND DAMPER BAR LENGTH ")
1005  FORMAT(2X,"ENTER THE STATOR-ROTOR BORE,THE MEAN END RING AND ROTOR
     1 DIAMETER"/)
1006  FORMAT(2X,"ENTER NUMBER OF STATOR SERIES TURNS AND STATOR SLOTS PE
     1R POLE PER PHASE"/)
1007  FORMAT(2X,"ENTER STATOR POLE SLOT AND TOOTH WIDTHS AND STATOR SKEW
     1"/)
1008  FORMAT(2X,"ENTER STATOR WINDING PITCH,ROTOR POLE PITCH AND ARC"/)
1009  FORMAT(2X,"ENTER THE NUMBER OF DAMPER BARS PER POLE HEAD ")
1010  FORMAT(2X,"IF THE SPACING BETWEEN DAMPER BARS IS UNIFORM ENTER A .
     11.,IF NOT ENTER A .2. ")
1011  FORMAT(2X,"ENTER THE PHYSICAL ANGLE BETWEEN TWO DAMPER BARS (DEGRE
     1ES) ")
1012  FORMAT(2X,"ENTER",I2," NUMBERS CORRESPONDING TO THE",I2," DAMPER B
     1AR PITCHES"/)
1013  FORMAT(2X,"ENTER GAP UNDER POLE CENTER, MAXIMUM AND MINIMUM GAPS"/
     1)
1014  FORMAT(2X,"ENTER A .1. IF ROUND DAMPER BAR, ENTER A .2. IF NOT ")
1015  FORMAT(2X,"ENTER DAMPER BAR SLOT OPENING WIDTH AND DEPTH ")
1016  FORMAT(2X,"ENTER DAMPER BAR SLOT AND SLOT OPENING WIDTHS ")
1017  FORMAT(2X,"ENTER DAMPER BAR SLOT OPENING DEPTH, ANGLED SECTION OF
     1DAMPER BAR SLOT DEPTH",/,3X,"DEPTH OF DAMPER BAR IN DAMPER BAR SLO
     2T, AND DEPTH OF DAMPER BAR"/)
1018  FORMAT(2X,"ENTER DAMPER BAR AND END RING CROSS-SECTIONAL AREA"/)
      IF(NSP.NE.1) GO TO 602
      IF(NB.NE.(NDD*2))GO TO 601
      DO 599 N=1,NDD
599   TRN(N)=(2*FLOAT(N)-1)*ALP*TA/PI
```

```
      GO TO 602
  601 DO 598 N=1,NDD
  598 TRN(N)=2*FLOAT(N)*ALP*TA/PI
  602 CONTINUE
      IF(ABS(GMA-GMI).LT. .000001) NSIN=1
      ANDD=FLOAT(NDD)
CALCULATION OF THE ADD FACTOR
      IF(NB.NE.(NDD*2)) GO TO 198
      IF(NSP.NE.1) GO TO 197
      ADD=SIN(ANB*ALP)/(ANB*SIN(ALP))
      CBD=(ANB/2.)*(1.-ADD)
      CBO=(ANB/2.)*(1.+ADD)
      GO TO 196
  197 DO 200 N=1,NDD
  200 ADD=ADD+COS(TNN(N)*PI/TA)
      ADD=ADD/ANDD
      CALL ALPHA(ADD,NB,NDD,ALP)
      CBD=ANDD*(1.-ADD)
      CBO=ANDD*(1.+ADD)
      GO TO 196
  198 IF(NSP.NE.1) GO TO 194
      ADD=(SIN(ALP*ANB)-SIN(ALP))/((ANB-1)*SIN(ALP))
      CBD=ANDD*(1.-ADD)
      ADD=SIN(ALP*ANB)/((ANB-1)*SIN(ALP))
      CBO=ANDD*(1.+ADD)
      ADD=ADD*(ANB-1)/ANB
      GO TO 196
  194 DO 201 N=1,NDD
  201 ADD=ADD+COS(TNN(N)*PI/TA)
      ADD=(ADD+.5)/ANDD
      CALL ALPHA(ADD,NB,NDD,ALP)
      CBO=ANDD*(1.+ADD)
      ADD=ADD-.5/ANDD
      CBD=ANDD*(1.-ADD)
  196 CONTINUE
      ADD=SIN(ANB*ALP)/(ANB*SIN(ALP))
```

```
CALCULATION OF THE SLOT LEAKAGE
      IF(NRD.NE.1) GO TO 298
      SLR=(.66+HB2/BB2)*(DLH/DL)
      GO TO 297
298   SLR=(HB4/BB1+2*HB2/(BB1+BB2)+HB1/(3*BB1)+HB2/BB2)*(DLH/DL)
297   CONTINUE
      GE=((TAA/TA)**2+GMA/GMI-1)*GA/(TAA/TA)**2
      AK1=SLR+TA/(12.*GE*ANB)+2*ANB*(DLB-DLH)/(3*PH*DL)
      SIG=(2./PI)*(ATAN(BS/(2.*GA))-(GA/BS)*(ALOG(1+(BS/(2*GA))**2)))
      FTTD=0.2284+0.0796*GA/BS-0.25*(BS/GA)*(1-SIG)
      FTTQ=0.2164+0.3183*(BT/BS)**0.5
      AK2D=(0.12*PI*ANB*DE)/(PH*DL*PD)*(2.+(COS(ANB*ALP)-ADD*COS(ALP))/(
     11.-ADD))+DLH*FTTD/DL
      ALDD=0.4*PI*(AK1+AK2D)
      AK2D=(0.12*PI*ANB*DE)/(PH*DL*PD)*(PI*(1-COS(ANB*ALP))/(ANB*ALP*(1+
     1ADD))+(COS(ANB*ALP)-ADD*COS(ALP))/(1.+ADD))+DLH*FTTQ/DL
      ALDQ=0.4*PI*(AK1+AK2D)
      CD1=(ALPB+SIN(ALPB*PI)/PI)/(4*SIN(ALPB*PI/2.))
      CQ1=CD1*(ALPB*PI-SIN(ALPB*PI))
      CD1=CD1*(ALPB*PI+SIN(ALPB*PI))
CALCULATION OF EQUIVALENT DAMPER BAR RESISTANCE
      RBD=(DLB/AB+TNNT/AR)*4.38/(10.**10)
      RBQ=(DLB/AB+TA/AR-TNNT/AR)*4.38/(10.**10)
      AKD=SIN(PI/(2*PH))/(XS*SIN(PI/(2*PH*XS)))
      AKD=AKD*SIN(W*PI/(TA*2.))
      IF(CHI.GT. .00001) AKD=AKD*SIN(CHI*PI/(2.*DLH))/(CHI*PI/(2.*DLH))
      CA=4*(XN*AKD)**2/PD
CALCULATION OF FINAL EQUATIONS
      RBD=RBD*CA/CBD
      RBQ=RBQ*CA/CBQ
      XLDD=CA*DL*2.*PI*F*ALDD/(10.**8*CBD)
      XLDQ=CA*DL*2.*PI*F*ALDQ/(10.**8*CBQ)
      XADD=CA*DL*F*0.8*PI*D*CD1/(10.**8*GE)
      XADQ=CA*DE*F*0.8*PI*D*CQ1/(10.**8*GE)
      XDDD=XLDD+XADD
      XDDQ=XLDQ+XADQ
```

```
      WRITE 1029
      WRITE 1030,XLDD,XADD,XDDD,RBD
      WRITE 1031,XLDQ,XADQ,XDDQ,RBQ
1029  FORMAT(1X,/,/,14X,"LEAKAGE",8X,"MUTUAL",8X,"SELF",/,13X,"REACTANCE
     1",5X,"REACTANCE",5X,"REACTANCE",5X,"RESISTANCE",/)
1030  FORMAT(2X,"DIRECT",4(3X,F11.9),/)
1031  FORMAT(2X,"QUADRA",4(3X,F11.9),/,/)
      STOP
      END
C**********************************************
      SUBROUTINE ALPHA(ADD,NB,NDD,ALP)
C**********************************************
      ALP=0.
      DEL=.1
      ANB=BNB=FLOAT(NB)
      IF(NB.NE.(2*NDD)) BNB=BNB-1
   50 ALP=ALP+DEL
      ATD=SIN(ANB*ALP)/(BNB*SIN(ALP))
      IF(ABS(ATD-ADD).LT..00001) RETURN
      IF(ATD.GT.ADD) GO TO 50
      ALP=ALP-DEL
      DEL=DEL/10
      GO TO 50
      END
```

Appendix C

Conversion of the Reference of the Equations from the Stator to the Rotor

(Ref 6:171-175)

Equations in the body of this thesis were developed with reference to the stator, but there are times when it is necessary to know the reactance and resistance of the damper windings referred to the rotor. Using the equivalent winding turns ratio of the rotor over the stator, the reference of the equations can be converted to the rotor. This ratio is

$$\frac{m_{Dd}(N_{Dd}\,k_{dw})^2}{m\,(N\,k_d\,k_p\,k_s)^2} \tag{112}$$

where k_d is the stator distribution factor Eq (66)

k_{dw} is the rotor winding factor

k_p is the stator pitch factor Eq (67)

k_s is the stator skew factor Eq (68)

N is the number of stator series turns per pole per phase

N_{Dd} is the number of rotor series turns per pole per phase (number of rotor circuits) Eq (90) or (94)

m is the number of stator phases

m_{Dd} is the number of rotor phases (2-- 1 direct and 1 quadrature)

For equations referred to the rotor, the factor C_a, from Eq (86), becomes

$$C_a = \frac{8(N_{Dd}k_{dw})^2}{mP} \tag{113}$$

The rotor winding factor is similar to the stator winding factor and uses identical development for the rotor distribution and pitch factors. In this development, it is assumed there is no rotor winding

skew, although if necessary, it could be included. The rotor distribution factor is

$$K_{dr} = \frac{\sin\left(N_{Dd}\frac{\alpha_b}{2}\right)}{N_{Dd}\sin\left(\frac{\alpha_b}{2}\right)} \tag{114}$$

where α_b is the average electrical angle between damper bars. The pitch factor, for the direct axis, is given by

$$k_{pr} = \sin\left(N_{Dd}\frac{\alpha_b}{2}\right) \tag{115}$$

and for the quadrature axis is

$$k_{pr} = \cos\left(N_{Dd}\frac{\alpha_b}{2}\right) \tag{116}$$

Thus the rotor winding factor for the direct axis is

$$k_{wr} = \frac{\sin^2\left(N_{Dd}\frac{\alpha_b}{2}\right)}{N_{Dd}\sin\left(\frac{\alpha_b}{2}\right)} \tag{117}$$

and for the quadrature axis

$$k_{wr} = \frac{\sin\left(N_{Dd}\frac{\alpha_b}{2}\right)\cos\left(N_{Dd}\frac{\alpha_b}{2}\right)}{N_{Dd}\sin\left(\frac{\alpha_b}{2}\right)} \tag{118}$$

Using Eq (113) to replace Eq (86) will refer the damper winding reactances and resistances to the rotor.

Vita

Richard E. Moore was born on 16 April 1954 in Schenectady, New York. He graduated from North Allegheny High School in Pittsburgh, Pennsylvania in 1972. After spending freshman year at Norwich University, Vermont and completing his undergraduate work at Grove City College, Pennsylvania in 1976, he received the Bachelor of Science Degree with concentration in Electrical Engineering and a commission in the United Stated Air Force. He entered the School of Engineering, Air Force Institute of Technology, as his initial active duty assignment in September 1976.

Permanent Address: 1949 Red Coach Road
Allison Park, PA 15101

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A set of equations for calculating the reactance and resistance of synchronous machine damper winding is developed and presented. The final equations are in terms of machine geometries, allowing the user to calculate impedances without knowing fluxes, induced currents or voltages.

The derivation of this set of equations is based on the method developed by M. E. Talaat. In this derivation, equations for induced currents and voltages are developed, and then reduced to impedance ratios in terms of machine geometries only. From these ratios the impedances are found.

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